**Forming and Solving Equations**

### Section A

1. Millie is also thinking of a number. She wants to be original, and after multiplying her number by 2 and adding 4, she divides by her *original* number. When she does so, she gets 3.
   1. Define a variable for Millie’s original number.
   2. Write down an expression for what she has after multiplying her number by 2 and adding 4.
   3. Divide this expression by the variable for the original number, and make it equal to 3.
   4. Solve the equation to find her number
2. Oliver is thinking of a number. He multiplies it by 5, subtracts 26, and then divides by 6. He then multiplies his original number by 3, and gets the same answer!
   1. Define a variable for Oliver’s number.
   2. Write an expression for what he has after multiplying by 5, subtracts 26, and then divides by 6.
   3. Write a separate expression for what he gets when he multiplies his number by 3.
   4. These are the same, so make an equation using them.
   5. Solve the equation to find Oliver’s number.
3. I think of a number, multiply it by 2 and then subtract 5 from the answer. If I start with the same initial number, but this time add 8 and then divide the result by 4, I find that I end up with the same result in both cases.
   1. Define a variable for the number I first thought of.
   2. write down an expression for each sum that I do.
   3. write down an equation from this story and solve it to find the number I thought of.
4. I think of another number, double it and then add 8 to the answer. If I start with the same original number, but this time subtract it from 2 and then double the result, I find that I end up with the same final result in both cases. What is my number?
5. Troy is thinking of a number. He multiplies it by 4 and divides by 5, then subtracts 3, and realizes that this number is 8 greater than his original number! What was Troy’s number?
6. James is thinking of a number. First, he multiplies his number by 10 and adds 4. Then he multiplies his original number by 6 and adds 12. Much to his surprise, he finds he gets the same number each time! What was his original number?
7. Olivia is thinking of a number. She multiplies her number by 4, subtracts 8, then multiplies by 3. Finally she subtracts this value from 30. Her final answer is a number that is 7 less than her original number. What was her original number?
8. Jake is thinking of a number. He starts by subtracting his number from 2; he multiplies this value by 3; finally he subtracts this value from the number which is ten times as great as his original number. Lo and behold his answer is precisely equal to his original number! What was it?
9. Evie is thinking of a number. She notes that if you divide the number 3 greater than her number by the number 3 less than her number, you get 7. What is her number?
10. Louis is thinking of a triangle. In his triangle the lengths of the three sides are consecutive integers (eg. 7cm, 8cm and 9cm, or 15cm, 16cm and 17cm). The perimeter of the triangle is 42cm. What is the length of the smallest side?
11. Make up your own “thinking of a number” question.

**Section B**

1. Four identical lumps of gold on one side of some scales balance with one identical lump and a 22kg mass.
   1. Copy and complete: *“Let x be the \_\_\_\_\_\_\_\_\_ of each \_\_\_\_\_\_\_”*
   2. Form an equations using this variable
   3. Solve your equation to find the mass of each lump of gold
   4. Substitute this mass back into your original equation to check that it is correct.
2. Sana buys a tin of paint, and pours the contents into a tub. She adds 15 litres which she finds in her garage, and now has three times as much paint as she started with.
   1. Form an equation for the litres of paint in each tin.
   2. Solve your equation to work out how many litres of paint were in each tin.
3. James knows that he will get the same amount of sweets from two packets plus fifteen extra sweets or five packets plus three extra sweets. Form and solve an equation to find how many sweets there are in each packet.
4. Tom had eight bags of boiled sweets. He then gave twelve sweets to his brother, and was left with six bags of sweets. Form and solve an equation to work out how many were there in a bag.
5. Albert’s age next year will be twice his age last year.
   1. Define a variable for Albert’s current age.
   2. write down an expression for Albert’s age next year
   3. write down an expression for Albert’s age last year.
   4. write down an expression for twice Albert’s age last year. Remember brackets.
   5. write down an equation and solve it to find Albert’s current age.
6. I share a box of sweets with my friend. We each get the same number. She gives me 7 of her sweets because she hates liquorice. I now have eight times more sweets then she does. Write down an equation from this story and solve it to solve the problem.
7. A gambler has some money. If he were to lose £6 and then double this new amount he would have the same amount of money as if he won £10.50 and then lost half of this new amount. How much does he have?
8. Ed has some money. He spends £7.50 on a Justin Timberlake CD and now has one third of what he started with. How much did he have to begin with?
9. Two football teams, Highgate United and Hampstead Allstars, notice that midway through the season Highgate had scored one and a half times more goals than Hampstead. In the next four matches, they each score a total of 12 goals and now Hampstead have three quarters of the goals that Highgate have. Work out
   1. how many goals Highgate had to start with.
   2. how many goals Hampstead finished with.
10. A school bus is full as it sets off. At the first stop 3 children get off and at the next stop a third of the remaining children get off, leaving the bus only half full. How many children were on the bus initially?

**Section C**

All of these problems can be solved using algebra.

You may wish to try to work out how to use algebra yourself, in which case you should just try them. If you want some hints as to how to use algebra, look at the next page.

**Problem A**

20cm

A large rectangle is made up of 5 identical smaller rectangles, as shown. The width of the rectangle is 20 cm. What is the area of the large rectangle?

**Problem B**

In a sequence of positive integers, every term after the first two terms is the sum of the two previous terms in the sequence. If the fifth term is 2004, what is the maximum possible value of the first term?

|  |  |  |
| --- | --- | --- |
| 7 |  | 8 |
|  |  |  |
| 12 |  | 13 |

**Problem C**

What is the sum of the five numbers which must be placed in the empty cells of this magic square so that every row, every column and both diagonals have the same total?

**Problem D**

Three identical rectangular cards can be placed end to end (with their short sides touching) to make a rectangle A, and can be placed side by side (with their long sides touching) to make rectangle B. The perimeter of rectangle A is twice the perimeter of rectangle B. Find the ratio of the length of the short side to the length of the long side of each card.

# Hints

**Problem A**

20cm

A large rectangle is made up of 5 identical smaller rectangles, as shown. The width of the rectangle is 20 cm. What is the area of the large rectangle?

Let the width of each small rectangle be x

1. Draw a picture and mark on x around the where it applies around the edge of the rectangle.
2. Using the 20cm, write an expression for the length of each rectangle
3. Add this expression to the lengths around the edge.
4. Write an expression for the length of the large rectangle using the bottom of it.
5. Write an expression for the length of the large rectangle using the bottom of it.
6. These two things must be the same – form an equation using this information.
7. Solve your equation to find the width of the small rectangle.
8. Hence work out the area of the large rectangle

**Problem B**

In a sequence of positive integers, every term after the first two terms is the sum of the two previous terms in the sequence. If the fifth term is 2004, what is the maximum possible value of the first term?

Let the first number be x

1. What is the most sensible choice of the second number?
2. Work out the third, fourth and fifth terms, in terms of x.
3. Use this to form and solve an equation.
4. Note: this shouldn’t quite work – you may need to change your second term.

|  |  |  |
| --- | --- | --- |
| 7 |  | 8 |
|  |  |  |
| 12 |  | 13 |

**Problem C**

What is the sum of the five numbers which must be placed in the empty cells of this magic square so that every row, every column and both diagonals have the same total?

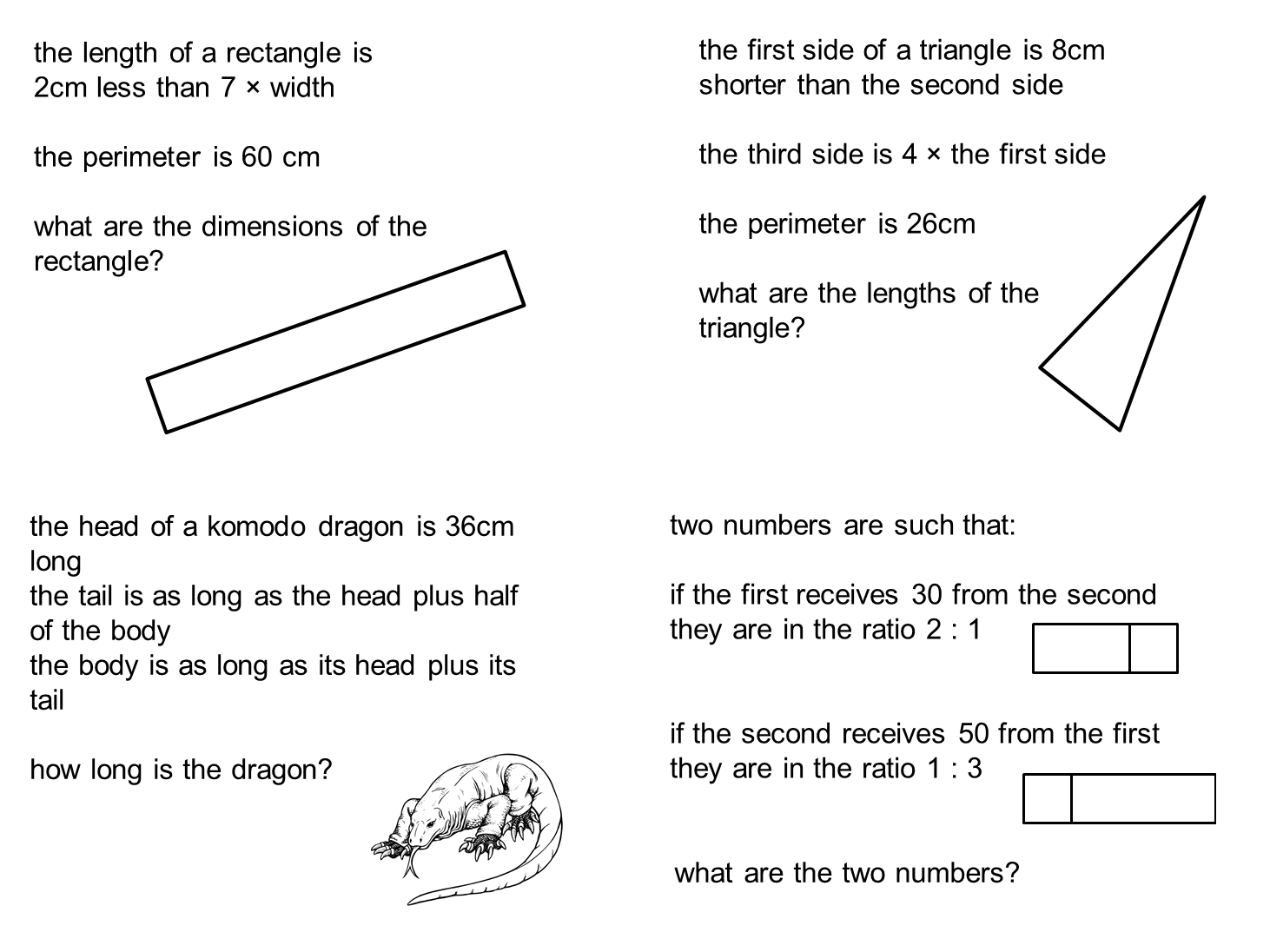
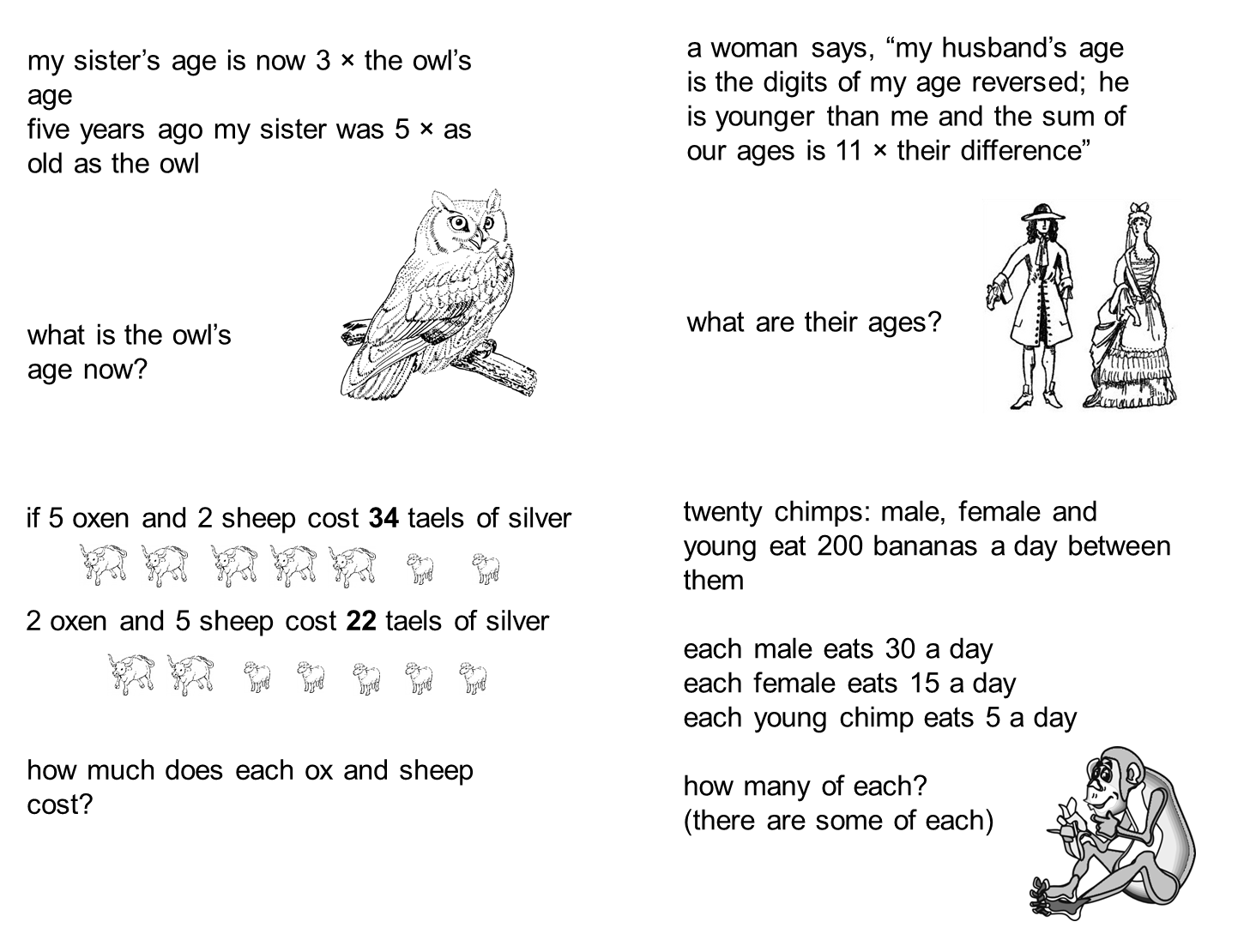
Let the number in the centre be x.

1. What is the magic “total” (the sum of every column and diagonals)
2. Write an expression for the empty cell between the 7 and the 8
3. Write an expression for the empty cell between the 12 and 13.
4. Form an equation using the middle column.
5. Solve your equation to find x, then complete the magic square.
6. Use this to work out the sum of the five mystery numbers

**Problem D**

Three identical rectangular cards can be placed end to end (with their short sides touching) to make a rectangle A, and can be placed side by side (with their long sides touching) to make rectangle B. The perimeter of rectangle A is twice the perimeter of rectangle B. Find the ratio of the length of the short side to the length of the long side of each card.

Let x be the length of the short side and y be the length of the long side.

1. Write an expression for the perimeter of rectangle A
2. Write an expression for the perimeter of rectangle B
3. Using the fact that perimeter A = 2 x perimeter B, form an equation.
4. Simplify this equation to find the ratio between the sides.
5. 
6. 

# Forming and Solving Equations - Homework

For each question you must **define a variable** before **forming an equation**, then **solving it.**

1. I buy a box of ball bearings and pour them all into a box. I then pour in 90 more ball bearings. There are three times as many ball bearings in the box than there were in the box that I bought.
   1. Copy and complete: *“Let x be the number of \_\_\_\_\_ \_\_\_\_\_\_\_\_ in a \_\_\_\_\_\_\_\_”*
   2. Form an equation to represent this situation
   3. Solve your equation to work out how many ball bearings there are.
2. I share a box of sweets with my friend. We each get the same number. She gives me 7 of her sweets because she hates liquorice. I now have eight times more sweets then she does. If b is the number of sweets we each get at first,
   1. write down an expression for the number of sweets I end up with.
   2. write down an expression for eight times the number of sweets my friends end up with.
   3. write down an equation from this story and hence find the number of sweets in the box by solving your equation
3. James has some Maltesers and then goes and buys another 18 Maltesers. He now has four times as many as he started with.
   1. Define a variable in this situation.
   2. Form and solve an equation to work out how many he started with.
4. I had seven bottles of fizzy orange. Having had six 80ml glasses of fizzy orange, I now have three bottles left. How much fizzy orange was there in each bottle?
5. \*I buy 12 balls of string. After using 5 metres of string to make an abacus I have only eight tenths of the string left. How much string was on each ball?

Answer box:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 120 |  | 45 | 6 | 9 |

**Mark scheme**

|  |  |  |  |
| --- | --- | --- | --- |
| **5** | None of below | **P** | Less than half questions correct |
| **4** | Most completed, Method shown | **S** | Around half questions correct |
| **3** | All completed, Answers self-marked | **G** | Nearly all questions correct |
| **2** | Variables all defined fully “let x be…” | **V** | All questions correct |
| **1** | Bonus questions attempted | **O** | Bonus questions correct |