## Trig A

Motivation: We have found a link between the sides of a triangle (Pythagoras) and we know a link between the angles (they add to 180), so now we would like something which links the sides to the angles.

Indirect Prerequisites: Scale factors, similar triangles, right-angled triangles.

Definition: The Tangent scale factor (TSF) is what you multiply to get from the Adjacent to the Opposite sides of a triangle. This fits in with the scale-factor theme around this point in the scheme – percentages, direct proportion.

First, pupils calculate the tangent scale factors in triangles – no angles involved.

Then, find the tangent scale factor for several triangles, all of which have the same angle. What do you notice? The TSF is always the same.

Why is this? A good extension question, afterwards to be explained as a group – the triangles are similar, so the scale factors within them do not change (if you double the adjacent and double the opposite, the TSF stays the same).

So, for this given angle (eg. ), we know that the TSF will always be 3/2 or 1.5. You can then use this to find the missing sides in other triangles with an angle of . A little bit of practice of this.

Another option is to do this in a less guided way: give pupils a sheet of triangles which are all (or mostly) similar, with the same angle marked. On some triangles, give both the Adjacent and the Opposite. On others, just give one of these. Make it so the scale factors between the triangles are messy, but within the triangles are nice. Higher-attaining pupils will probably automatically invent and use the TSF.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| angle | 31° | 35° | 42° | 56° | 58° | 61° | 69° | 71° | 76° |
| tangent s.f. | 0.6 | 0.7 | 0.9 | 1.5 | 1.6 | 1.8 | 2.6 | 2.9 | 4.0 |

The question may arise, what if the angle is different? Then the TSF will be different. Fortunately, someone has calculated them for us. Here is table of some of them:

Using this table, pupils can find the missing sides, and then angles (they’re probably need a couple of example of each type) of triangles. All non-calculator, good practice of fractions and decimals.

Extensions can include more applied questions, such as how tall is the tree if…

## Trig B

Motivation: We have found a link between the opposite, adjacent and the angles in a triangle. What about the hypotenuse? We could always use the TSF + Pythag, but that is a bit of a drag. It will also turn out that the new scale factors we invent have other uses.

Definitions: The Sine Scale Factor is what you multiply to get from the Hypotenuse to the Opposite sides of a triangle. The Cosine Scale Factor is what you multiply to get from the Hypotenuse to the Adjacent sides.

Introduce these individually in the same was as the tangent scale factor, only this time should be quicker as they are more familiar with the approach.

Then discuss how to decide which scale factor to use, followed by examples and questions in which all three scale factors are used and pupils have to choose the appropriate one.

If pupils are struggling to remember which scale factors use which sides, you could give them SHO CHA TAO – an kind of eastern-sounding version of the more traditional mnemonic.

Extensions: show that you get the same answer using one scale factor vs the ‘wrong’ scale factor and Pythagoras.

## Trig C

A scientific calculator stores (or in reality, can work out – I don’t actually know this for certain, but I suspect so!) the tables.

Start with some practice of filling in tables using the calculator: they’ll need both the original buttons, and the ‘shift’ versions when given a scale factor and asked for the angle. You could leave pupils to figure this out for themselves.

Then some examples and practice of how this is written formally (because you don’t want to draw a table every time) out.

Q: Find the tangent scale factor for the angle of A:

Q: Find the angle which has a cosine scale factor of A:

We don’t write it because that could be confused with . Your calculator is unfortunately designed by a careless mathematician / there isn’t enough space to write arcsin!

We apply this to answer the same (type of) problems from Trig B – Decide which scale factor is appropriate, but this time use your calculator to find the value of that scale factor.

First, finding sides only, then finding angles, then mixed up, probably with examples in between each section.

Extension: can pupils see any patterns in the links between the angle and the scale factors?

## Trig D

Indirect prerequisites: Measuring angles, Solving Equations, Significant Figures

At this point we insist on pupils writing out their solutions in a formal manner. For example:

This is as a chance for pupils to apply the trig rules to solve problems in which they have to use more than one scale factor and where they have to apply them to solve a problem in context.

I personally think that this should include the classic “surveying” problem: we know our current altitude, how can we work out the altitude of a nearby object (in Geneva – ideally a mountain).

Also mix in questions which require Pythagoras.

Extensions: can pupils see any patterns in the links between the angle and the scale factors?

How are the three scale factors linked?

## Trig E

Indirect Prerequisites: Plotting Graphs

Here we come up with an answer as a group to the suggested extension problem in the previous two sections, so it would be good to give pupils some time to investigate this at the start: can pupils see any patterns in the links between the angle and the scale factors? Now, provide some scaffolding: perhaps a table of angle against each of the scale factors and a question prompting how we can represent pairs of variables such as this?

Once a few pupils have proposed drawing a graph, provide some axes and ask them to plot each of the scale factors against the angles.

How can we extend these graphs? Clearly a right-angled triangle can’t have another angle of or larger, but we can look at what happens to a right-angled triangle inscribed within a unit circle as the angle grows.

There is a nice Geogebra file which demonstrates this smartly.

## Trig F

Indirect Prerequisites: Pythagoras, Describing Angles using letters, Drawing Plan and Elevations from 3d objects.

Place letters in the 8 vertices of the room.

As the pupils to identify right-angles and describe them using the letters. Keep going, there are many.

Then give them the length, width and height of the room. Start with a right angled triangle on the floor of the room: How can we work out the length of the diagonal across the floor (distance from A to C, for example)? How can we use that to work out the length of the diagonal from top corner to opposite bottom corner? How could we have solved this problem in a different way.

Also finding angles in these triangles, or lengths given angles: full recap of trigonometry, writing out solutions formally. Apply to other solids such as prisms and pyramids.

Revise volumes and surface areas of these solids: possible links to science include crystal structures and rates of reaction.

Extensions: speed of ascent up a pyramid where speed up a slope of angle