**Proof**

### Counter-Examples

Below are some conjectures. Disprove them all.



**Example** Everybody in this class is female.

1. All numbers are even.
2. All primes are odd.
3. No even numbers are divisible by odd numbers.
4. All square numbers are bigger than two.
5. All sixth form students at MCS take mathematics A-level.
6. No number less than three is divisible by three.
7. All integers have a rational square root.
8. No cubes may be written as the sum of two squares.

### Proof by Algebra

If the following conjectures are true, prove them directly. If they are false, find a counter-example.

**Examples**

The product of an even number and an odd number is even.

Every prime number is the sum of two square numbers.

A four-digit number formed by writing down two digits and then reversing them is divisible by 11.



1. The product of two odd numbers is odd.
2. A four-digit number formed by writing down two digits and then repeating them is divisible by 101.
3. The sum of the squares of any three consecutive integers is an odd number.
4. If S is an integer
   1.  is a square number
   2.  is a triangle number (given by )
5. The sum any four consecutive integers is divisible by 4.
6. For any two real numbers  and , 
7. For any pair of numbers  and ,  is the sum of two squares.
8. If p is a prime number greater than 3, then either p+1 or p-1 is divisible by 6.
9. If x is a number such that x+1 or x-1 is divisible by 6, then x is a prime.
10. If x is divisible by 6, then at least one of x+1 or x-1 is a prime number.

### Proof by Contradiction

**Examples** There is no largest number.  is an irrational number.

The sum of a rational number and an irrational number is always irrational.

1. There is no smallest positive number.
2. If  is an integer and is odd, then  is odd.
3. The product of a rational number and an irrational number is always irrational.
4. If  is an integer and is a multiple of 3, then  is a multiple of 3.
5.  is irrational.
6. There are infinitely many prime numbers.
7. \*Every even number greater than two is the sum of two primes.

### Proof by…

**Odds and Evens**

1. The sum of any two even numbers is always even.
2. The sum of an odd number and an even number is always odd.
3. The product of an odd number and an even number is always even.
4.  is an odd number for all values of *x*.
5. The product of two odd numbers is always odd.
6. The product of two even numbers is a multiple of 4.
7. If *x* is even, then  is odd.

**Exhaustion**

1. No square number ends in an 2.
2. No even number ends in a 3.
3. No cube of an odd number ends in 4.

**Cases (subset of Exhaustion)**

1. Every integer that is a perfect cube is a multiple of 9, or is 1 more than a multiple of 9, or is 1 less than a multiple of 9.
2. Prove that if n is an integer, then  is even.
3. Prove that if n is any integer which is not divisible by 5, then  leaves a remainder of 1 or 4 when it is divided by 5.
4. Prove that, if is a real number, then

**Factorising**

1. If is an integer, is always divisible by 3.
2. If is an integer, is always divisible by 2.
3. If is an integer, is always divisible by 6.

**Diagram**

1. 
2. The triangle with sides of length ,  and  is right angled.
3. Pythagoras’ Theorem: 
4. A point on the perpendicular bisector of A and B is equidistant from A and B.